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Inform on Algebra

Algebra in Lecce

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AN INTRODUCTION TO GALOIS CONNECTIONS

A Galois connection is a specific kind of correspondence between two ordered sets. More precisely, if $(P, \leq)$ and $(Q, \leqslant)$ are ordered sets, a pair of maps $({}^{\triangleright}, {}^{\triangleleft})$, where ${}^{\triangleright} : P \to Q$ and ${}^{\triangleleft} : Q \to P$, is called a *Order Preserving Galois connection* if the following propriety holds

$$\forall p \in P \;\; \forall q \in Q \quad p^{\triangleright} \preceq q \iff p \leq q^{\triangleleft}$$

Conversely $({}^{\triangleright}, {}^{\triangleleft})$ is called a *Order Reversing Galois connection* if the following propriety holds

$$\forall p \in P \;\; \forall q \in Q \quad p^{\triangleright} \preceq q \iff q^{\triangleleft} \leq p$$

The most famous Galois connection is the one obtained by Galois himself.
Let E be a separable extension of a field C. Then we can consider the sets

$$I(C, E) = \{C \subseteq D \subseteq E \mid D \text{ subfield of } E\}$$

$$Gal(E|C) = \{\alpha \in Aut(E) \mid \forall c \in C \; \alpha(c) = c\},$$

Respectively, the set of all extension of C in E, and the Galois group of E with respect to C. It can be proven that the following function

$$\phi : (I(C, E), \subseteq) \to (\mathcal{L}(Gal(E|C)), \leq) \quad D \mapsto Gal(E|D)$$

Is an order reversing isomorphism between the lattice $\mathcal{L}(Gal(E|C))$, the subgroups of the Galois group of E with respect to C, and the set of extensions of C in E, ordered by inclusion. The pair (ϕ, ϕ^{-}) is then an order reversing Galois connection between the two lattices.

In this seminar we will introduce a more general theory of Galois connections and a strong link between them and complete lattices. With this important result in mind we will first introduce a few lattice theoretic concepts, before returning to the theory of Galois connections to prove our initial result.

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