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Dottorato in Matematica e Informatica XXXV Ciclo

Discussione Tesi

27 settembre 2023 Ore 15:30

Aula "F5"

Dipartimento di Matematica e Fisica "E. De Giorgi"

Model Reduction and Data-Driven techniques for spatio-temporal dynamics of Turing and oscillatory type

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The numerical approximation of RD-PDE systems whose solutions present spatio-temporal dynamics of Turing and oscillatory type might be very challenging, because high computational times and memory occupation are tipically required to yield accurate discrete solutions.
We investigate a suitable application of Model Order Reduction (MOR) and data-driven techniques to reduce the computational costs, while preserving the features of the spatio-temporal dynamics. In particular, we apply the Proper Orthogonal Decomposition (POD) with a Galerkin projection, the Discrete Empirical Interpolation Method (DEIM) for an hyper-reduction of the nonlinear terms and the Dynamic Mode Decomposition (DMD).
We show that solutions of surrogate models built by classical POD exhibit an unstable error behaviour over the dimension of the reduced space. To overcome this drawback, first of all, we propose a POD-DEIM technique with a correction term and its adaptive version that accounts for the peculiar dynamics due to Turing instability. We show the effectiveness of the proposed methods in terms of accuracy and computational costs for a selection of RD systems, with increasing degree of nonlinearity and more structured patterns (like labyrinths).
Among data-driven methods, we consider the DMD algorithm and we show that its original version does not provide accurate reconstructions for datasets describing oscillatory dynamics or spatio-temporal Turing-Hopf instability. Inspired from the classical "divide and conquer" approach, we propose a piecewise version of DMD (pDMD) to overcome this issue. Numerical experiments show that very accurate reconstructions can be obtained by pDMD.
The stabilization of POD and DMD, the accuracy and computational efficiency gained by the new techniques introduced, encourage their application for future developments, in particular for energetic applications, like: i) smart parameter identification techniques to compare numerical solutions with experimental data, (ii) economic construction of PDE model-based training sets for Machine Learning techniques.

Main References:

[1]- B. Bozzini, A. Monti, I. Sgura - Model-reduction techniques for PDE models with Turing type electrochemical phase formation dynamics, Applications in Engineering Science, Vol. 8 (2021) 100074, doi.org/10.1016/j.apples.2021.100074
[2] - A. Alla, A. Monti, I. Sgura - Adaptive POD-DEIM correction for Turing pattern approximation in reaction-diffusion PDE systems, Journal of Numerical Mathematics (JNUM), De Gruyter Eds, 31(3):205–229 (2023), ISSN 15702820, DOI 10.1515/jnma-2022-0025
[3]- A. Alla, A. Monti, I. Sgura - Piecewise DMD for oscillatory and Turing spatio-temporal dynamics, arXiv preprint arXiv:2303.06512, (2023).